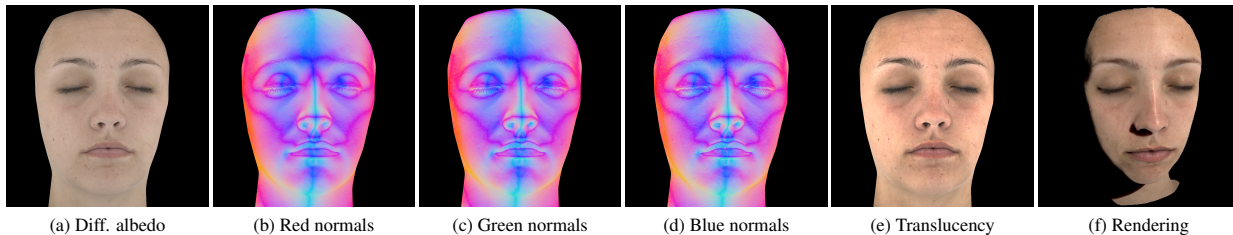# Estimating Diffusion Parameters from Polarized Spherical Gradient Illumination

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(a) Diff. albedo   (b) Red normals   (c) Green normals   (d) Blue normals   (e) Translucency   (f) Rendering

Figure 1: *Estimating spatially varying diffusion parameters from polarized spherical gradient illumination. RGB translucency parameters (e) inferred from diffuse albedo (a) and RGB diffuse normals (b-d).*

Introduction. Accurately modeling and reproducing the appearance of real-world materials is crucial for the production of photoreal imagery of digital scenes and subjects. The appearance of many common materials is the result of subsurface light transport that gives rise to the characteristic "soft" appearance and the unique coloring of such materials. Jensen et al. [2001] introduced the dipole-diffusion approximation to efficiently model isotropic subsurface light transport. The scattering parameters needed to drive the dipole-diffusion approximation are typically estimated by illuminating a homogeneous surface patch with a collimated beam of light, or in the case of spatially varying translucent materials with a dense set of structured light patterns. A disadvantage of most existing techniques is that acquisition time is traded off with spatial density of the scattering parameters.

Polarized Spherical Gradients. Recently, Ma et al. [2007] proposed a technique to obtain high quality estimates of diffuse and specular albedo and photometric normal maps from just eight photograph under four different polarized spherical gradient lighting conditions. In addition, Ma et al. also proposed a hybrid normal rendering technique that approximates the soft appearance of subsurface scattering with local shading using measured RGB diffuse normals. This suggest a connection between spherical gradient illumination and subsurface scattering.

In this work, we aim to formalize this apparent connection between subsurface scattering parameters and observations under spherical gradient illumination of translucent materials based on radiative transfer theory [Ishimaru 1978]. In particular, we show that *dense per-surface-point* scattering parameters can be *directly* obtained from observations under spherical gradient illumination (cross-polarized to discard specular reflections), *without* resorting to any explicit fitting of observed scattering profiles.

Background. Light transport in highly scattering translucent materials can be well approximated by diffusion theory [Ishimaru 1978; Jensen et al. 2001]. According to radiative transfer theory, diffusion can be accurately approximated by a two-term spherical harmonic expansion of radiance:

$$L(x, \omega) = \frac{1}{4\pi}\phi(x) + \frac{3}{4\pi}\omega \cdot \vec{E}(x), \qquad (1)$$

where $\phi(c)$ is the scalar fluence and $\vec{E}(x)$ is the vector irradiance. Substituiting Equation 1 in the radiative transfer equation and assuming semi-infinite material, leads to the well-known diffuse BSSRF [Jensen et al. 2001]:

$$R_d(r) = -D\frac{(\vec{n} \cdot \nabla\phi(x_o))}{d\Phi_i(x_i)}, \qquad (2)$$

where $r = ||x_o - x_i||$, and $D = 1/3\sigma_t'$ is the diffusion constant.

(a) Diff. albedo   (b) Translucency   (c) Rendering
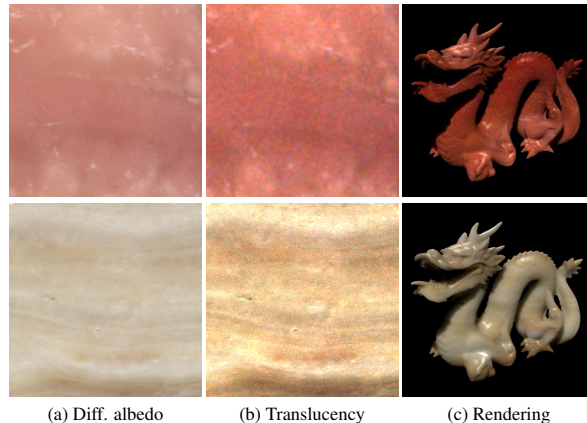
Figure 2: *Spatially varying diffusion parameters of material samples estimated using spherical gradient illumination. Top-row: Red wax. Bottom-row: Polished marble.*

Diffusion from Gradients. Relating Equations (1) and (2) yields a mechanism for estimating scattering parameters of dipole diffusion from observations of the 1st-order spherical gradients. Specifically, a BRDF approximation of Equation 2 relates the observed diffuse albedo R_d (cross-polarized 0th order spherical statistics) to the normal aligned component of the estimated diffuse normal $\vec{n}$ (cross-polarized 1st order spherical statistics [Ma et al. 2007]) via the diffusion constant D. Assuming that the surface normal is along the +Z direction, this leads to the following compact relation between the diffusion constant and polarized spherical gradients:

$$D \propto \frac{R_d}{|\vec{n}_z|}. \qquad (3)$$

Conclusion. Equation 3 provides a mechanism for *directly* obtaining *dense spatially-varying* diffusion parameters from just four observations of translucent materials under polarized spherical gradient illumination.

References

ISHIMARU, A. 1978. *Wave Propagation and Scattering in Random Media*. Academic Press, New York.

JENSEN, H. W., MARSCHNER, S. R., LEVOY, M., AND HANRAHAN, P. 2001. A practical model for subsurface light transport. In *Proceedings of ACM SIGGRAPH 2001*, 511–518.

MA, W.-C., HAWKINS, T., PEERS, P., CHABERT, C.-F., WEISS, M., AND DEBEVEC, P. 2007. Rapid acquisition of specular and diffuse normal maps from polarized spherical gradient illumination. In *Rendering Techniques*, 183–194.