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Robustness of Multiple Listener Equalization With Magnitude Response Averaging

Sunil Bharitkar[1], Philip Hilmes[1], and Chris Kyriakakis[1]

[1]*Immersive Audio Laboratory, Integrated Media Systems Center, University of Southern California, 3740 McClintock Avenue, Los Angeles, CA 90089-2564.*

Correspondence should be addressed to Sunil Bharitkar (`bharitka@usc.edu`)

ABSTRACT
Traditionally, room response equalization is performed to improve sound quality at a given listener. However, room responses vary with source and listener positions. Hence, in a multiple listener environment, equalization may be performed through spatial averaging of magnitude responses at locations of interest. However, the performance of averaging based equalization, at the listeners, may be affected when listener positions change. In this paper, we present a statistical approach to map variations in listener positions to a performance metric of equalization for magnitude response averaging. The results indicate that, for the analyzed listener configurations, the zone of equalization depends on distance of microphones from a source and the frequencies in the sound.

1. INTRODUCTION

A typical room is an acoustic enclosure that can be modeled as a linear system whose behavior at a particular listening position is characterized by an impulse response. The impulse response yields a complete description of the changes a sound signal undergoes when it travels from a source to a receiver (microphone/listener). The signal at the receiver consists of direct path components, discrete reflections that arrive a few milliseconds after the direct sound, as well as a reverberant field component. In addition, it is well established that room responses change with source and receiver locations in a room [1], [2]. In other words, a room response can be uniquely defined by a set of spatial co-ordinates $l_i \triangleq (x_i, y_i, z_i)$. This assumes that the source is at origin and the receiver i is at the spatial co-ordinates, x_i, y_i and z_i, relative to a source in the room.

Due to variations in room responses with listener positions relative to a source, in a multiple listener environment, room equalization should be performed for all listeners present in the room. With a good multiple listener equalization technique, all listeners in a given environment will experience high quality sound. Furthermore, the equalization technique should be robust to variations in listener head movements. Specifically, the equalized response should not vary significantly in the vicinity of the listeners.

One method for providing simultaneous multiple listener equalization is by measuring the room responses with microphones at all possible listener positions, averaging the measurements, and inverting the stable component of the result. The microphones are generally positioned, during measurements, at the expected center of a listener head. Although this equalization is aimed at achieving uniform frequency response coverage for all listeners, its performance is often limited due to, (i) mismatch between microphone measurement location and actual location for the center of the listener head, or (ii) variations in listener locations (e.g., head movements).

In this paper, we propose a statistical approach using modal equations for evaluating the robustness of equalization based on magnitude response averaging, due to the introduction of variations in room responses (generated either through (i) or (ii)), for different listener arrangements relative to a fixed sound source. In Section 2, we introduce necessary background used in the development of the proposed robustness analysis. Section 3 is dedicated to the development of the robustness analysis for spatial average based equalization. In Section 4, we present results based on simulations for different listener arrangements relative to a fixed source. Section 5 concludes the paper.

2. ROOM ACOUSTICS FOR SIMPLE SOURCES

The Green's function derived from the wave theory for sound fields in an enclosure is given by [1], [3]

$$\begin{aligned}
p_{\omega,avg} &= \frac{1}{N}\sum_{l=1}^{N} |p_\omega(\underline{q}_l)| \\
p_\omega(\underline{q}_l) &= jQ\omega\rho_0 \sum_{\underline{n}} \frac{p_{\underline{n}}(\underline{q}_l)p_{\underline{n}}(\underline{q}_o)}{K_{\underline{n}}(k^2 - k_{\underline{n}}^2)} \quad (1) \\
\underline{n} &= (n_x, n_y, n_z); k = \omega/345; \underline{q}_l = (x_l, y_l, z_l) \\
k_{\underline{n}} &= \pi[(\frac{n_x}{L_x})^2 + (\frac{n_y}{L_y})^2 + (\frac{n_z}{L_z})^2]^{1/2} \\
& \int\int_V\int p_{\underline{n}}(\underline{q}_l)p_{\underline{m}}(\underline{q}_l)dV = K_{\underline{n}}; (\underline{n} = \underline{m}) \\
& = 0 (\underline{n} \neq \underline{m})
\end{aligned}$$

where the eigenfunctions $p_{\underline{n}}(\underline{q}_l)$ can be assumed to be orthogonal to each other under certain conditions, and the point source being at $\underline{q}_o$.

For a rectangular enclosure with dimensions (L_x, L_y, L_z), $\underline{q}_o = (0,0,0)$, the eigenfunctions and K_n in (1) are

$$\begin{aligned}
p_{\underline{n}}(\underline{q}_l) &= \cos(\frac{n_x\pi x_l}{L_x})\cos(\frac{n_y\pi y_l}{L_y})\cos(\frac{n_z\pi z_l}{L_z}) \\
p_{\underline{n}}(\underline{q}_o) &= 1 \\
K_{\underline{n}} &= \int_0^{L_x}\cos^2(\frac{n_x\pi x_l}{L_x})dx \int_0^{L_y}\cos^2(\frac{n_y\pi y_l}{L_y})dy \\
& \int_0^{L_z}\cos^2(\frac{n_z\pi z_l}{L_z})dz \\
&= \frac{L_xL_yL_z}{8} = \frac{V}{8} \quad (2)
\end{aligned}$$

3. ROBUSTNESS ANALYSIS OF EQUALIZATION USING MAGNITUDE RESPONSE SPATIAL AVERAGING

A performance function, $W_\omega^{(i)}(\epsilon)$, that is used for analyzing the robustness, of spatial average equalization, to room response variations is given as

$$\begin{aligned}
W_\omega^{(i)}(\epsilon) &= E\{|p_\omega(\nu_\epsilon^{(i)})p_{\omega,avg}^{-1} - p_\omega(\underline{q}_i)p_{\omega,avg}^{-1}|^2\} \quad (3) \\
p_{\omega,avg} &= \frac{1}{N}\sum_{l=1}^{N}|p_\omega(\underline{q}_l)|
\end{aligned}$$

where, $p_\omega(\nu_\epsilon^{(i)})$ is the pressure at locations in the ϵ neighborhood of position i having pressure $p_\omega(\underline{q}_i)$ (ϵ-neighourhood is defined as all points at a distance of

ϵ from location i), and $E\{.\}$ denotes the expectation operator.

The performance measure (1) is defined in such a manner that when the displacement ϵ, about position i (whose response $p_\omega(\underline{q}_i)$ is originally used for determining the spatially averaged equalization filter $p^{-1}_{\omega,avg}$), is zero, then $W^{(i)}_\omega(\epsilon) = 0$. Furthermore, the performance measure is computed as an average of the error between the response at the equalized location and the response at a displaced location having distance ϵ from the equalized location. Obviously, this is not a psychoacoustic measure (i.e., it is not computed with different weights at different frequencies). However, a future direction would be to form a psychoacoustically motivated composite measure using a weighted combination of $W^{(i)}_\omega(\epsilon)$ over specific frequencies ($f \in$ [20 Hz, 20 kHz]).

For simplicity, in our analysis, we assume variations in responses due to displacements (or mismatch) in a horizontal plane (x-y plane). The analysis can be easily extended to include the vertical plane. Thus, simplification of (3) leads to

$$\begin{aligned} W^{(i)}_\omega(\epsilon) &= N^2/(\sum_{l=1}^{N} |p_\omega(\underline{q}_l)|)^2 * \\ & \quad [\overbrace{E\{p_\omega(\nu^{(i)}_\epsilon) p^*_\omega(\nu^{(i)}_\epsilon)\}}^{I} \\ &- \overbrace{E\{p^*_\omega(\nu^{(i)}_\epsilon)\}}^{II} p_\omega(\underline{q}_i) \\ &- \overbrace{E\{p_\omega(\nu^{(i)}_\epsilon)\}}^{III} p^*_\omega(\underline{q}_i) \\ &+ \overbrace{|p_\omega(\underline{q}_i)|^2}^{IV}] \end{aligned} \quad (4)$$

We only need to compute the statistics associated with Terms (I), (II) and (III) (the terms within the expectations) in (4), since Term (IV) is a deterministic quantity.

Now, $E\{p_\omega(\nu^{(i)}_\epsilon) p^*_\omega(\nu^{(i)}_\epsilon)\}$ is the average over all locations along a circle of radius ϵ from the i-th listener location. Assuming the source, all listeners, and each of the listener displacements are along the same z-plane ($z = 0$), then (I) in (4) can be simplified using the following equations,

$$p_\omega(\nu^{(i)}_\epsilon) = \frac{j8Q\omega\rho_0}{V} \sum_{\underline{n}} \frac{\cos(\frac{n_x\pi\phi^{(i)}_x}{L_x})\cos(\frac{n_y\pi\phi^{(i)}_y}{L_y})}{(k^2 - k^2_{\underline{n}})} \quad (5)$$

$$E\{p_\omega(\nu^{(i)}_\epsilon) p^*_\omega(\nu^{(i)}_\epsilon)\} = |\psi_1|^2 \sum_{\underline{n},\underline{m}} (1/\psi_2)\psi_3 \quad (6)$$

$$\begin{aligned} \psi_1 &= \frac{8Q\omega\rho_0}{V} \\ \psi_2 &= (k^2 - k^2_{\underline{n}})(k^2 - k^2_{\underline{m}}) \\ \psi_3 &= E\{\cos(\frac{n_x\pi\phi^{(i)}_x}{L_x})\cos(\frac{n_y\pi\phi^{(i)}_y}{L_y})\cos(\frac{m_x\pi\phi^{(i)}_x}{L_x}) \\ & \quad \cos(\frac{m_y\pi\phi^{(i)}_y}{L_y})\} \end{aligned} \quad (7)$$

Now, with $\phi^{(i)}_x = x_i + \epsilon\cos\theta$ and $\phi^{(i)}_y = y_i + \epsilon\sin\theta$

$$E\{\cos(\frac{n_x\pi\phi^{(i)}_x}{L_x})\cos(\frac{n_y\pi\phi^{(i)}_y}{L_y})\cos(\frac{m_x\pi\phi^{(i)}_x}{L_x})\cos(\frac{m_y\pi\phi^{(i)}_y}{L_y})\} =$$

$$\begin{aligned} \frac{1}{2\pi}\int_0^{2\pi} & \cos(\frac{n_x\pi(x_i+\epsilon\cos\theta)}{L_x})\cos(\frac{n_y\pi(y_i+\epsilon\sin\theta)}{L_y}) \\ & \cos(\frac{m_x\pi(x_i+\epsilon\cos\theta)}{L_x})\cos(\frac{m_y\pi(y_i+\epsilon\sin\theta)}{L_y})d\theta \end{aligned} \quad (8)$$

Eq. (8) can be solved using the Matlab *trapz* function. However, we found an approximate closed form expression to be computationally much faster. The following expressions were derived from standard trignometric formulae; and using the first two terms in the polynomial expansion of the cosine function, and the first term in the polynomial expansion of the sine function since $(\epsilon/L_x, \epsilon/L_y, \epsilon/L_z) << 1$. Thus,

$$\begin{aligned} E\{\cos(\frac{n_x\pi\phi^{(i)}_x}{L_x})\cos(\frac{n_y\pi\phi^{(i)}_y}{L_y})\cos(\frac{m_x\pi\phi^{(i)}_x}{L_x})\cos(\frac{m_y\pi\phi^{(i)}_y}{L_y})\} \\ = \frac{1}{2\pi}(A + B + C) \end{aligned} \quad (9)$$

where

$$\begin{aligned} A &= \pi\cos(\frac{n_x\pi x_i}{L_x})\cos(\frac{n_y\pi y_i}{L_y})\cos(\frac{m_x\pi x_i}{L_x})\cos(\frac{m_y\pi y_i}{L_y}) * \\ & \quad [2 - \epsilon_y^2 v_y - \epsilon_x^2 v_x + \frac{3}{4}(\epsilon_x^4 u_x^4 + \epsilon_y^4 u_y^4) - \\ & \quad \frac{1}{8}(\epsilon_x^2\epsilon_y^4 u_y^2 v_x + \epsilon_x^4\epsilon_y^2 u_x^2 v_y) + \frac{1}{4}\epsilon_x^2\epsilon_y^2 v_x v_y + \frac{3}{64}\epsilon_x^4\epsilon_y^4 u_x^2 u_y^2] \\ B &= \pi\epsilon_y^2 u_y \cos(\frac{n_x\pi x_i}{L_x})\cos(\frac{m_x\pi x_i}{L_x})\sin(\frac{n_y\pi y_i}{L_y})\sin(\frac{m_y\pi y_i}{L_y}) * \\ & \quad [2 - 0.5\epsilon_x^2(m_x^2 + n_x^2 - 0.5\epsilon_x^2 u_x^2)] \quad (10) \\ C &= \pi\epsilon_y^2\epsilon_x^2 u_x u_y \sin(\frac{n_x\pi x_i}{L_x})\sin(\frac{n_y\pi y_i}{L_y})\sin(\frac{m_x\pi x_i}{L_x})\sin(\frac{m_y\pi y_i}{L_y}) \\ & \quad +\pi\epsilon_x^2 u_x \sin(\frac{n_x\pi x_i}{L_x})\sin(\frac{m_x\pi x_i}{L_x})\cos(\frac{m_y\pi y_i}{L_y})\cos(\frac{n_y\pi y_i}{L_y}) * \\ & \quad [2 - 0.5\epsilon_y^2(n_y^2 + m_y^2 - 0.5\epsilon_y^2 u_y^2)] \end{aligned}$$

where,

$$\begin{aligned}\epsilon_x = \frac{\pi\epsilon}{\sqrt{2}L_x} \quad &; \quad \epsilon_y = \frac{\pi\epsilon}{\sqrt{2}L_y} \\ u_x = n_x m_x \quad &; \quad u_y = n_y m_y \\ v_x = (m_x^2 + n_x^2) \quad &; \quad v_y = (m_y^2 + n_y^2)\end{aligned} \tag{11}$$

Thus (10) can be substituted in (9) and subsequently in (4) to determine Term I.

Now Terms (II) and (III) in (4) can be combined to give,

$$-E\{p^*_\omega(\nu_\epsilon^{(i)})\}p_\omega(\underline{q}_i) - E\{p_\omega(\nu_\epsilon^{(i)})\}p^*_\omega(\underline{q}_i) =$$

$$-2|\frac{8Q\omega\rho_0}{V}|^2 \sum_{\underline{m},\underline{n}} \frac{p_{\underline{m}}(\underline{q}_i)E\{\cos(\frac{n_x\pi\phi_x^{(i)}}{L_x})\cos(\frac{n_y\pi\phi_y^{(i)}}{L_y})\}}{(k^2 - k^2_{\underline{m}})(k^2 - k^2_{\underline{n}})} \tag{12}$$

Now again using $\phi_x^{(i)} = x_i + \epsilon\cos\theta; \phi_y^{(i)} = y_i + \epsilon\sin\theta$, we have

$$E\{\cos(\frac{n_x\pi\phi_x^{(i)}}{L_x})\cos(\frac{n_y\pi\phi_y^{(i)}}{L_y})\} =$$

$$\frac{1}{2\pi}\int_0^{2\pi}\cos(\frac{n_x\pi(x_i + \epsilon\cos\theta)}{L_x})\cos(\frac{n_y\pi(y_i + \epsilon\sin\theta)}{L_y})d\theta \tag{13}$$

Thus, upon again using the fact that $(\epsilon/L_x, \epsilon/L_y, \epsilon/L_z) << 1$, we can solve (13) as

$$E\{\cos(\frac{n_x\pi\phi_x^{(i)}}{L_x})\cos(\frac{n_y\pi\phi_y^{(i)}}{L_y})\} = \frac{1}{2\pi}\cos(\frac{n_x\pi x_i}{L_x})\cos(\frac{n_y\pi y_i}{L_y})$$

$$*[2\pi - \pi(\epsilon_x^2 n_x^2 + \epsilon_y^2 n_y^2) + \frac{\pi}{4}\epsilon_y^2\epsilon_x^2 n_x^2 n_y^2] \tag{14}$$

Substituting (14) in (12) and subsequently into (4) gives Terms II and III.

4. RESULTS

Simulation of (3) using (6), (9), and (12) was performed in Matlab for a room of dimensions 6 m × 6 m × 6 m with two positions that were equalized with magnitude response averaging. The two positions were $\underline{q}_1 = (2,2)$ and $\underline{q}_2 = (3,4)$ (shown in Fig. 1 as asterisks), whereas the source was at the origin (shown in Fig. 1 as a circle at the origin).

Fig. 2 shows the results from the simulation for 0.1 m $\leq \epsilon \leq$ 0.5 m and f = 150 Hz. Clearly, the trend of poor equalization performance as ϵ increases is shown for both positions. An interesting difference, at higher displacements, can be seen between the two positions, with the farther position (relative to the source) $\underline{q}_2$ had a more robust equalization as compared to position $\underline{q}_1$. Future directions will focus on determining the cause of such differences.

5. CONCLUSIONS

In this paper we presented a statistical approach using modal equations for evaluating the robustness of equalization based on magnitude response averaging, due to the introduction of variations in room responses (generated either through (i) or (ii)), for different listener arrangements relative to a fixed sound source. The simulations were performed for a two “listener” setup with a simple source in a cubic room. For both listener positions, the equalization performance degraded with displacements as is to be expected. However, at higher displacements, the farther position (relative to the source) had a more robust equalization as compared to the closer position.

Future research will be directed towards, (i) determining the cause of the differences in performance measure, $W_\omega^{(i)}(\epsilon)$, between equalized positions, (ii) extending the simulations over more frequencies and positions, (iii) forming a psychoacoustically motivated composite measure using a weighted combination of $W_\omega^{(i)}(\epsilon)$ over specific frequencies,

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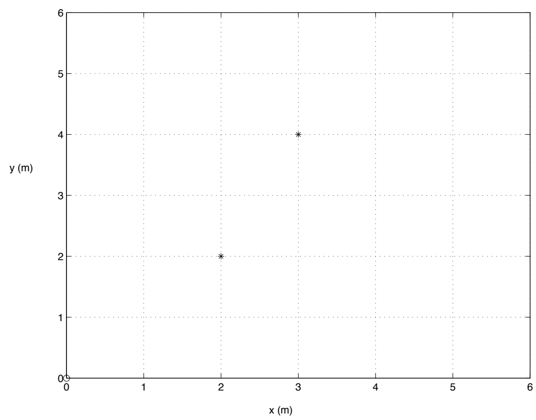

Fig. 1: Simulated setup for a two position robustness analysis. The two equalized positions are marked by an asterisk and the source is denoted by a circle.

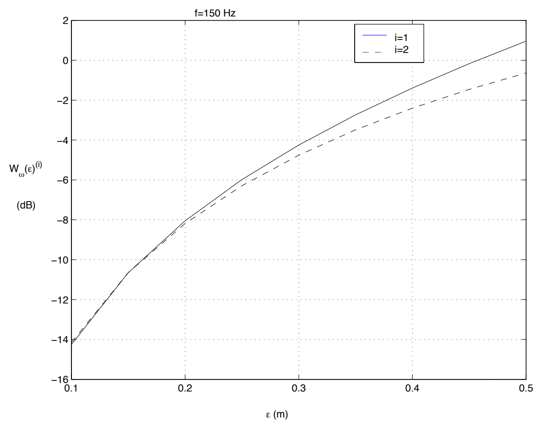

Fig. 2: Results from the simulation for the setup in Fig. 1. The solid line indicates the robustness of spatial average equalization at position 1, while the dashed line shows the robustness at position 2 .