

# **ROTORS for WIND POWER**

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# 1 ROTORS for WIND POWER

### 1.1 Classification of machines, that extract energy from the wind

There exists an abundance of all types of machines to convert wind power into useful mechanical power on a shaft. There is an enormous number of inventions, claiming to be superior to all other types, many of them being trash (Figure 1.1).

Machines to capture wind power can be classified according to the following aspects (see also Figure 1.2):

- \* orientation rotor - axis to wind direction:
  - parallel to the wind
  - perpendicular to the wind
- \* orientation with respect to ground:
  - horizontal axis
  - vertical axis
- \* dominant propelling factor:
  - lift
  - drag

\* movement:

- rotation

- translation (sailing ship)

- oscillation

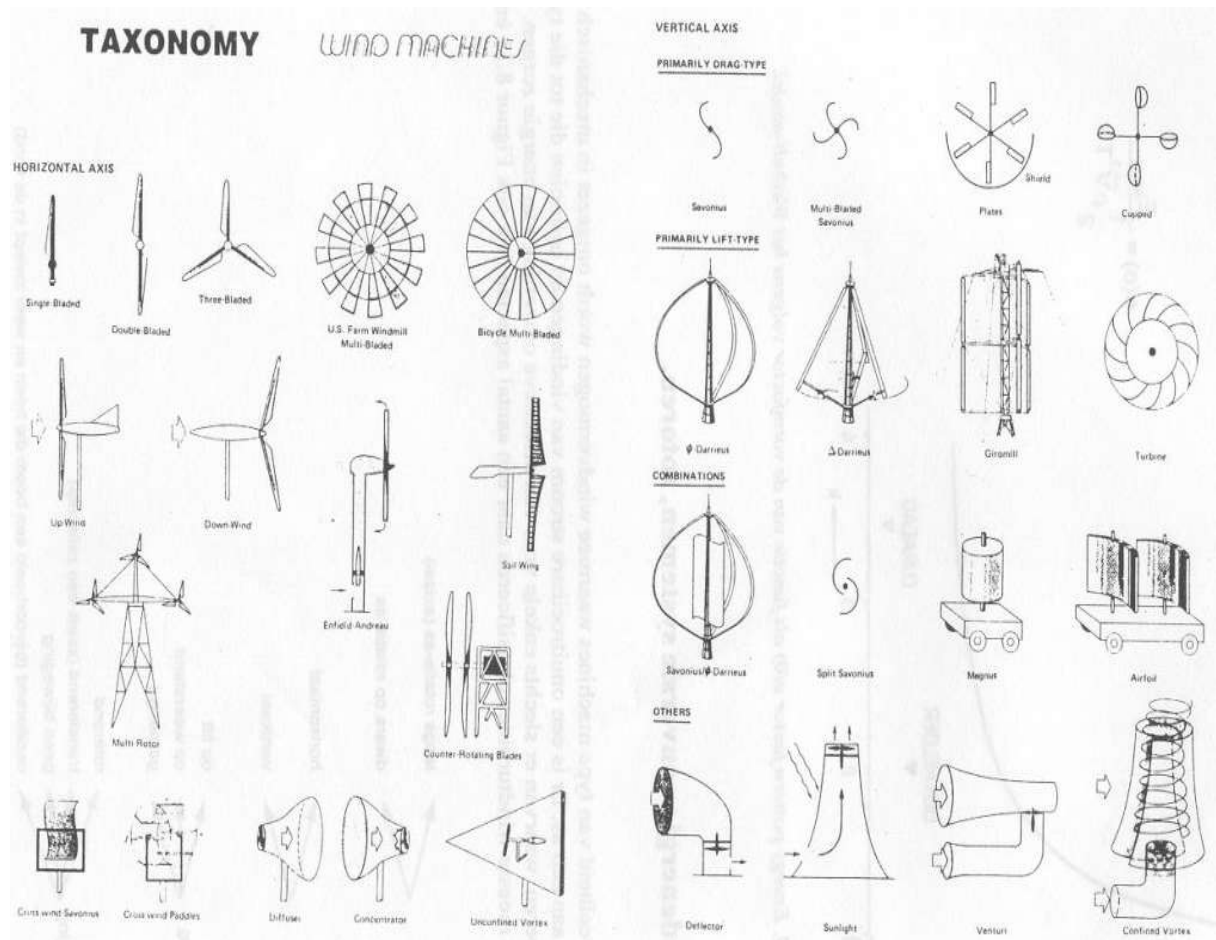


Figure 1.1 Types of machines to extract power from the wind. Plenty of crazy ideas!

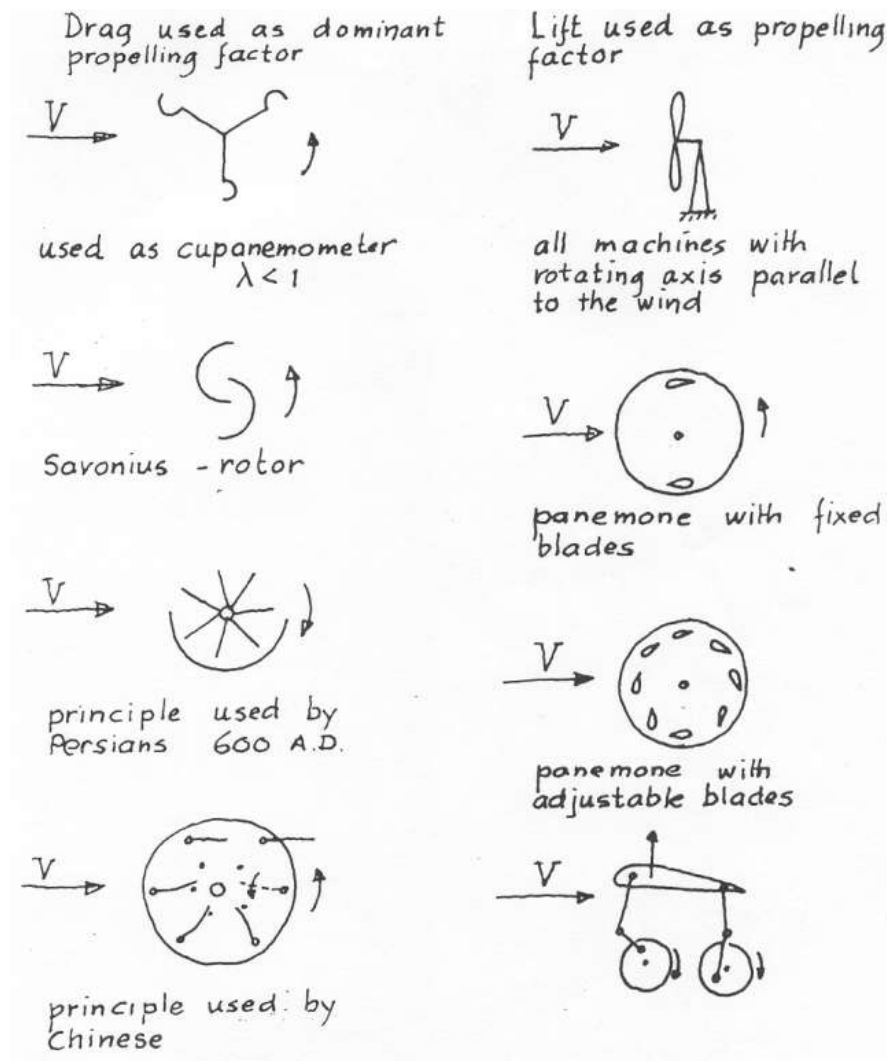


Figure 1.2 Classification of wind rotors using lift or drag as propelling factor

(The Chinese panemone uses lift also)

## 2 BASIC PRINCIPLES; AIRFOIL CHARACTERISTICS

Nearly all machines that extract energy from the wind use either LIFT or DRAG or both LIFT and DRAG as the propelling force.

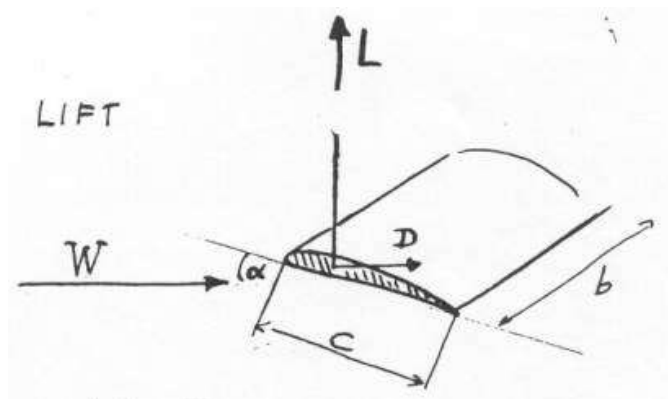
### 2.1 LIFT and DRAG

Any object that is placed in a uniform flow with velocity  $W$  experiences

\* a force perpendicular to  $W$ : LIFT

\* a force in the direction of  $W$ : DRAG

As an example let us take an airfoil, such as is used on airplanes. Let  $W$  be the relative velocity to the airfoil (Figure 2.1)



Lift perpendicular to  $W$

Drag in direction of  $W$

*Figure 2.1 Lift and drag*

An airplane is carried by the lift. There's no power needed to keep the airplane in the air, because the lift is perpendicular to the direction the plane is flying (in fact this is only true if the airfoil were infinitely long).

The power of the airplane motor is needed to move against the drag on the wings and of course the body.

It can be shown that we can write the drag and lift as follows:

$$L = C_L(\alpha) \cdot \frac{1}{2} \rho W^2 \cdot A \quad (2.1a)$$

$$D = C_D(\alpha) \cdot \frac{1}{2} \rho W^2 \cdot A \quad (2.1b)$$

in which A is the area c x b (see Figure 2.1). c is the so-called chord of the airfoil section,  $\alpha$  is the angle of attack.

$C_L(\alpha)$  is called: lift coefficient

$C_D(\alpha)$  is called: drag coefficient

$C_L(\alpha)$  and  $C_D(\alpha)$  are dimensionless;  $\frac{1}{2}\rho W^2$  has the dimension of pressure (N/m<sup>2</sup>);  $\rho$  is the density (in our case of air);  $\rho_{\text{air}} \approx 1.2 \text{ kg m}^{-3}$ .

Equations (1a) and (1b) are important. For one particular shape of an airfoil, we can determine  $C_L(\alpha)$  and  $C_D(\alpha)$  from experiments in a wind tunnel. Knowing A and  $\alpha$  and the wind tunnel velocity W, we can determine  $C_L(\alpha)$  and  $C_D(\alpha)$  by measuring the lift L and the drag D.

A typical example of how  $C_L$  and  $C_D$  vary with the angle  $\alpha$  is shown in Figure 2.2. It is useful to plot  $C_L$  versus  $C_D$ . The tangent determines an important point (A), with  $\alpha = \alpha_0$  (its optimum value), where  $C_D/C_L$  is minimum or  $C_L/C_D$  is maximum. This is a design point for airplanes, wind rotors etc. and of course gliders. Why?

Values of lift and drag coefficients can be found in different handbooks. Values of  $C_L$  for different airfoils at point A are around 1

( $C_L \approx 1$ ); ( $C_L/C_D$ )<sub>max</sub> values range from 20 to 150 (see also Figure 2.6). The corresponding angles of  $\alpha$  are 5 to 10°.

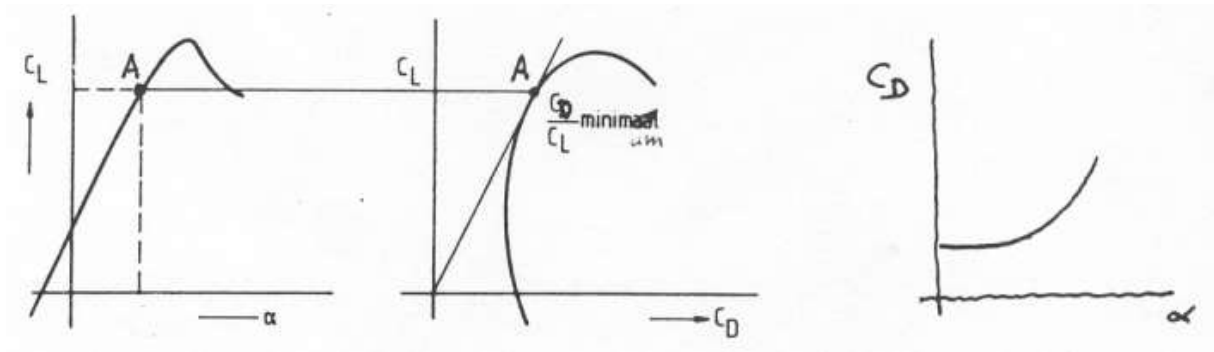


Figure 2.2 Lift and drag coefficient  $C_L(\alpha)$  and  $C_D(\alpha)$

To understand how lift is generated, let us look at the flow around an airfoil. At small angles  $\alpha$  the flow around the airfoil is smooth. The figures show the corresponding pressure distribution on the airfoil surface.

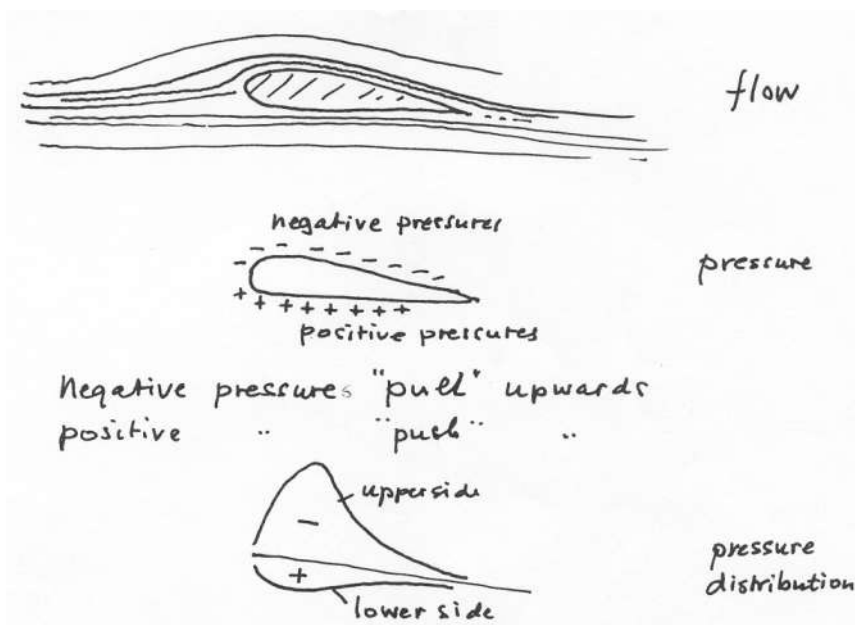


Figure 2.3 Flow pressure and pressure distribution around an airfoil at small angles of incidence  $\alpha$

The negative pressure on the top side "pulls" the airfoil upwards, the positive pressure on the lower side "pushes" it upwards. Note that the contribution of negative pressures to lift on the upper-side is larger than that of positive pressures on the lower side. If angle  $\alpha$  increases, the pressure distribution become more pronounced and lift increases, so  $C_L(\alpha)$  increases.

At a certain point (higher  $\alpha$ ) the flow breaks away from the upper side (see Figure 2.4). This is called **STALL**. The pronounced negative pressure distribution is destroyed and the lift drops.



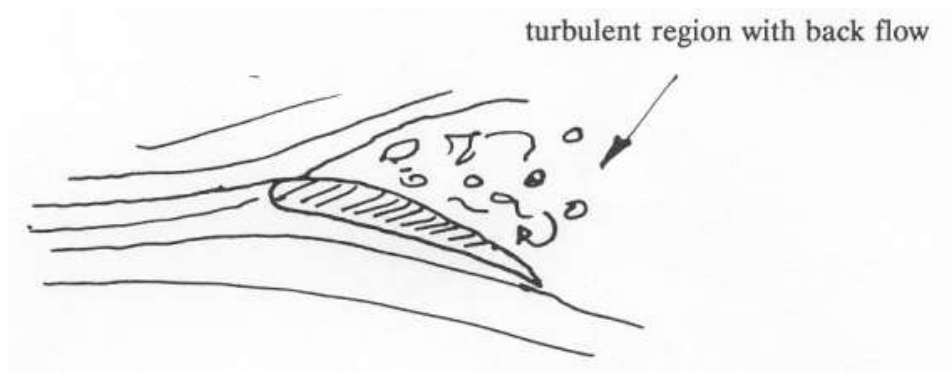


Figure 2.4 Flow around airfoil at stall conditions

## 2.2 The effect of the Reynolds number on airfoil characteristics

In reality it shows that the drag and lift coefficients are not only dependent on  $\alpha$ , but also on the so-called REYNOLDS NUMBER.

For an airfoil it is defined as follows:

$$Re = \frac{W \cdot c}{\nu}$$

$W$  is relative velocity (see Figure 2.1);  $c$  is the chord,  $\nu$  is the kinematic viscosity.

$\nu$  for air is approximately  $15 \cdot 10^{-6} \text{ m}^2/\text{s}$  (at room temperature).

Note that the Re-number is dimensionless!

Basically Reynolds number takes account of the viscous forces relative to other forces in the flow.

Fig. 2.5 shows the airfoil characteristics of the well-known NACA 4412-profile for different Reynolds numbers. Below a certain value of  $Re$  the characteristics change considerably (curve d and e compared to a, b and c).

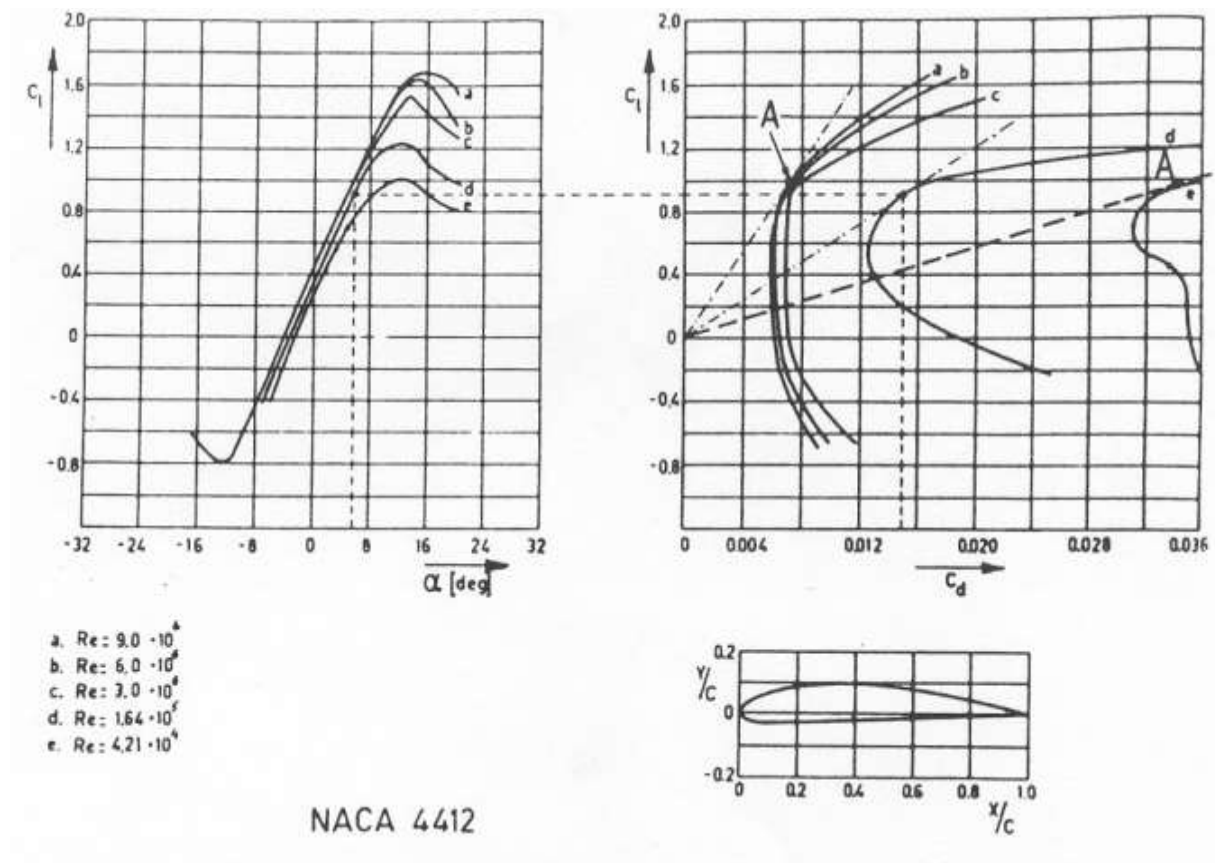


Figure 2.5 Lift and drag coefficient for NACA 4412 profile at different Reynolds numbers.

As we have seen the minimum value of  $(C_D/C_L)$  or the maximum value of  $C_L/C_D$  is an important value for designers. Figure 2.6 shows  $(C_L/C_D)_{\max}$  of all kinds of airfoils at different Reynolds numbers.

For airplanes:  $Re > 10^6$

For wind rotors:  $Re \geq 10^6$  for rotor diameters larger than 10 m.

Figure 2.6 Showing how  $(C_L/C_D)_{\max}$  varies with  $Re$  - number for a variety of profiles.  $(C_L/C_D)_{\max}$  corresponds to point A in Figure 2.2.  $C_L$  value at this point for different profiles is approximately:  $C_L \approx 1$ .

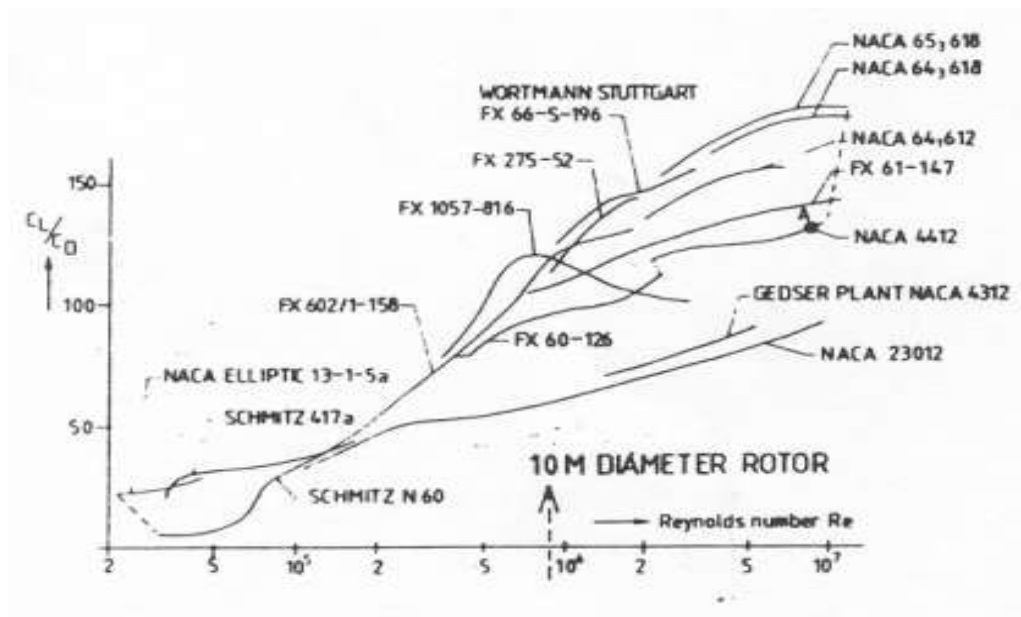


Fig. 2.6  $(C_L/C_D)_{\max}$  varies with  $Re$  - number for a variety of profiles.

Most airfoil data are available for high Reynolds numbers ( $Re > 10^6$ ). Less (but still many) for lower  $Re$ -numbers. Figure 2.6 shows that  $(C_L/C_D)_{\max}$  decreases with decreasing Reynolds number. In fact at Reynolds number around  $10^5$ , curved plates are quite good airfoils. For example, the Smitz 417a (usually called Göttingen 417) is a curved plate.

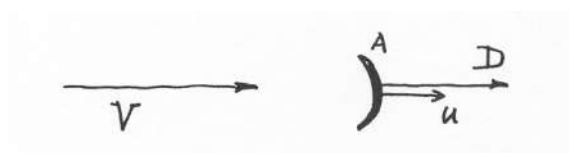
## 2.3 Whether to use lift or drag to extract energy from the wind?

It seems clear that we might try to extract energy from the wind by using either the drag force or the lift force. For a rotating machine this is rather complicated, so we will examine the question by analysing the movement in a straight line of a wing or sail, which is caused by either drag or lift.

### Extraction of energy from wind by a translating machine (e.g. a sailing boat)

#### a. Pure drag machine

Suppose a curved sail moves with a velocity  $u$  in the direction of the wind speed  $V$  as depicted below in Figure 2.8.



*Figure 2.8 Model of a pure drag machine.*

The drag force on the sail follows from equation (2.1b)

$$D = C_D \cdot 1/2 \rho (V - u)^2 A$$

as  $(V - u)$  is the velocity relative to the sail.

Power extracted by the sail:

$$P = D \cdot u = C_D \cdot 1/2 \rho V^3 (1 - u/V)^2 \cdot u/V \cdot A$$

Define a dimensionless speed ratio  $\lambda = u/V$

Power extraction by the sail is maximum if  $dP/d\lambda = 0 \rightarrow$

$$P_{\max} = 4/27 \cdot C_D \cdot 1/2 \rho V^3 \cdot A$$

for  $\lambda = u/V = 1/3$

Note that:  $u < V$ . So the maximum power is extracted at velocities of the sail lower than  $V$ . In fact, to extract power from the wind by a drag machine,  $u$  must be smaller than  $V$ .

**1. Simple lift machine with some drag (e.g. sailboat sailing perpendicular to the wind)**

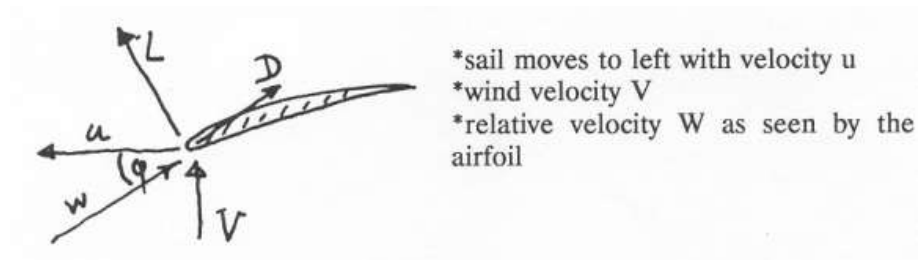


Figure 2.9 Model of a simple lift machine

\*  $L$  perpendicular to relative velocity  $W$

\*  $D$  in direction of relative velocity  $W$

Force component in direction of  $u$ :

$$F_u = L \sin \phi - D \cos \phi$$

(2.4)

Power extracted by "sail":

$$P = F_u \cdot u$$

The lift and drag are given by: (see Figure 2.1 and equation 2.1)

$$L = C_L \cdot \frac{1}{2} \rho W^2 \cdot c \cdot b$$

$c$  = chord

$b$  = length airfoil

$$D = C_D \cdot 1/2 \rho W^2 \cdot c \cdot b$$

$\lambda = u/V$  (speed ratio). Now Figure 2.9 shows  $\tan \phi = V/u = 1/\lambda$ .

We now find by substitution:

$$P = C_L \cdot 1/2 \rho V^3 \cdot \lambda \cdot \sqrt{1 + \lambda^2} \cdot (1 - C_D / C_L \cdot \lambda) \cdot c \cdot b$$

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P is maximum for  $dP/d\lambda = 0$ .

We find:

$$P_{\max} = C_L \cdot 1/2 \rho V^3 \cdot c \cdot b \cdot 4/27 \cdot (C_L / C_D)^2 \text{ for } \lambda = \frac{2 C_L}{3 C_D}$$

Now  $C_L \approx 1$  (see Figure 2.5 and 2.6)

Choose  $(C_L/C_D)_{\max}$  value.

If  $(C_L/C_D)_{\max} = 50 \rightarrow \lambda = 33.3 !!$  ; much higher than 1 !!

This shows that a relatively small, fast running airfoil can extract the power from the wind over larger areas! This is also the basic reason that a well-designed fast running one-bladed rotor can extract the same amount of energy from the wind as a slow running multi-bladed rotor. Further it is clear that the power we can extract with a lift machine is much higher than with a drag machine! (compare equations 2.3 and 2.4)

Note: 1) maximum  $C_D$  values: 2 for half cylinder

2)  $C_L$ -values at  $(C_L/C_D)_{\max}$  are approx.:  $C_L \approx 1$ .

In reality  $\lambda$ -values that can be reached are much lower. Not only the drag of the airfoil is important: the drag of the sailing boat in water is much higher.

A "sailing boat" on ice runs very fast (up to 100 km/hr). It has a low drag compared to a ship in water!

### Conclusion:

1. Always use lift as the driving propelling force. With the same area, lift machines can extract more power from the wind than a drag machine.
2. Drag machines operate at speeds lower than the wind speed. Lift machines can operate at higher speeds than the wind speed.
3. Always mistrust inventors with "drag" machines.

## 3 ROTATING MACHINES AND HORIZONTAL AXIS ROTORS

We will restrict ourselves to horizontal axis rotors. Basically the same principles are applicable to vertical axis rotors. These are however more complicated in analysing their aerodynamic characteristics. Before going into some aerodynamic aspects which determine the possible power extraction from the wind by a rotor, we will first describe some important overall performance characteristics and their definitions.

### 3.1 Rotor characteristics

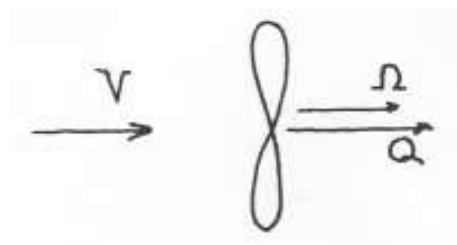


Figure 3.1 Wind power is converted by the rotor to mechanical shaft power

Wind power is converted into mechanical shaft power by the rotor (Figure 3.1). Rotor radius is  $R$ . Let shaft torque be equal to  $Q$ . Let  $n$  be number of revolutions per second of the shaft.  $\Omega = 2\pi.n$  is the angular shaft speed.

Mechanical power on the shaft is:

$$P = Q.\Omega \quad (3.1)$$

Define:

a) tip speed ratio:

$$\lambda = \frac{\Omega R}{V} \quad \text{noting that } \Omega R = V_{\text{tip}}, \text{ the tip speed of the rotor}$$

b) torque coefficient:

$$C_Q = \frac{Q}{1/2 \rho V^2 . \pi R^3}$$

c) power coefficient

$$C_P = \frac{P}{1/2 \rho V^3 \pi R^2}$$

A wind rotor extracts kinetic energy from the air flow. The velocity of the air behind the rotor is therefore lower than in front of it. The reaction force of the rotor on the airflow slows down the velocity of the air. So besides a torque, also an axial force is exerted on the rotor. In fact, this force decreases the momentum of the airflow. We call this axial force  $F_{\text{ax}}$  and define:

axial force coefficient:



$$C_{F_{ax}} = \frac{F_{ax}}{1/2 \rho V^2 \cdot \pi R^2}$$

To explain the meaning of the dimensionless coefficient, let us take an example of the rotor torque.

In Fig. 3.2 a plot is shown of the torque/rotational speed characteristics at different wind speeds, as could be measured in a wind tunnel. If we now convert these graphs into a graph of the dimensionless torque coefficient versus the tip speed ratio, we see that all the original curves merge into one single curve. This curve is called the dimensionless rotor torque characteristic of the wind rotor.

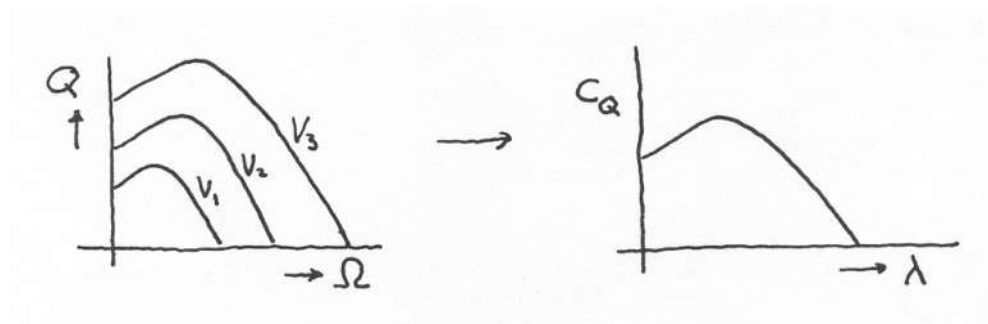


Figure 3.2 How different  $Q - \Omega$  curves merge into one single curve by writing them dimensionless

A similar result is found for the axial force and power curves.

Note that  $P = Q \cdot \Omega$ , so  $C_P = C_Q \cdot \lambda$

We will come back to rotor characteristics later on in section 4.

### 3.2 Simple aerodynamic aspects of rotor behaviour

There are two ways of trying to understand rotor behaviour:

1. by forgetting altogether that rotor blades are involved and replacing the rotor by a so-called actuator disk. This is an old concept dating back to the last century in trying to

understand ship propellers. Betz used the concept to determine the ideal maximum power extraction by a wind rotor (Betz ± 1926). It is normally referred to as “momentum theory”.

2. by looking at the forces on the individual rotor blades: “blade element theory”.

The two methods are used in combination to determine the rotor characteristics: so called “blade element/momentum theory”.

### 3.2.1 Betz model

Substitute the rotor by an actuator disk (see Figure 3.3). Assume that the disk extracts energy (in some way or other) from the airflow. The flow that goes through the disk loses energy and therefore slows down. Betz found that maximum energy is extracted if:

$$V_e = 1/3 V \text{ and } V_{disk} = 2/3 V.$$

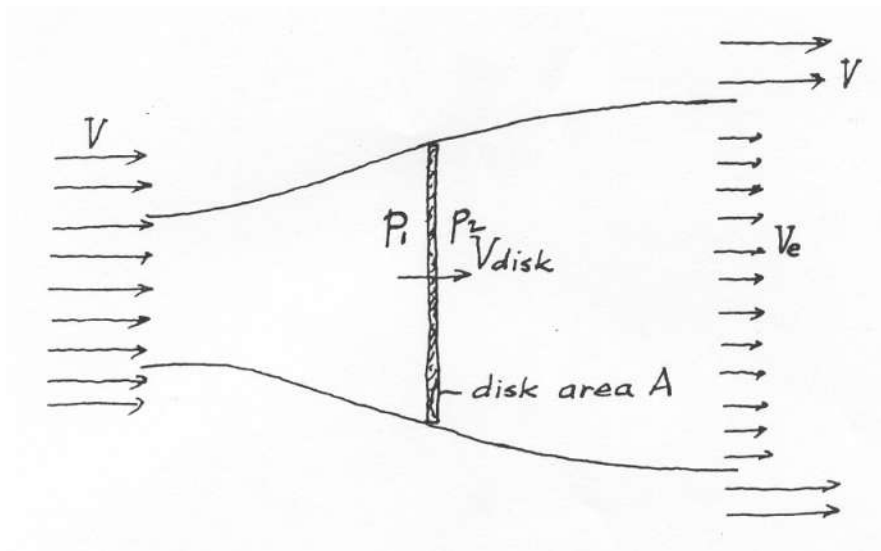
In that case:

$$P_{max.Betz} = 16/27 \cdot 1/2 \rho V^3 \cdot A \quad (3.7a)$$

$$\text{So } C_{Pmax} = 16/27 = 0.59 \quad (3.7b)$$

So approximately no more than 60% of the undisturbed wind power flowing through the equivalent disc area A can be extracted by a “machine”.

According to (3.7a) 8/9 Betz) 8/9 of the power of the air flow that goes through the disk (or rotor) is extracted by the disk, as the kinetic energy drops from  $1/2 \rho V^3$  to  $1/2 \rho V_e^3 = 1/9 \cdot 1/2 \rho V^3$ . But we see that a part of the flow through an area A upstream of the rotor is diverted outwards and does not pass through the rotor. This explains quantitatively Betz maximum. The power extraction is determined both by the amount of air that goes through the rotor and by the percentage of kinetic power that is extracted from it. By extracting little power, a large mass flows through the rotor, but with little power extracted. In the other extreme, if all the energy were extracted from the flow through the rotor, then the velocity behind the rotor would be zero! That means that all the flow would be diverted outside of the rotor plane. Betz optimum is a delicate balance between a sufficient airflow through the rotor and a sufficient power extraction from that air!



$$P_{max} = 16/27 \cdot 1/2 \rho V^3 \cdot A$$

Betz

$$\text{at } V_e = 1/3 V \text{ and } V_{disk} = 2/3 V$$

$$C_p = 16/27$$

Fig. 3.3 Description of a rotor as an actuator disk. Betz used this model to determine a theoretical limit of maximum power output of a rotor.

### 3.2.2 Blade element theory

As a blade element rotates, the velocity relative to it is different from the wind velocity itself. The speed of the blade element is  $\Omega r$  (Fig. 3.4).

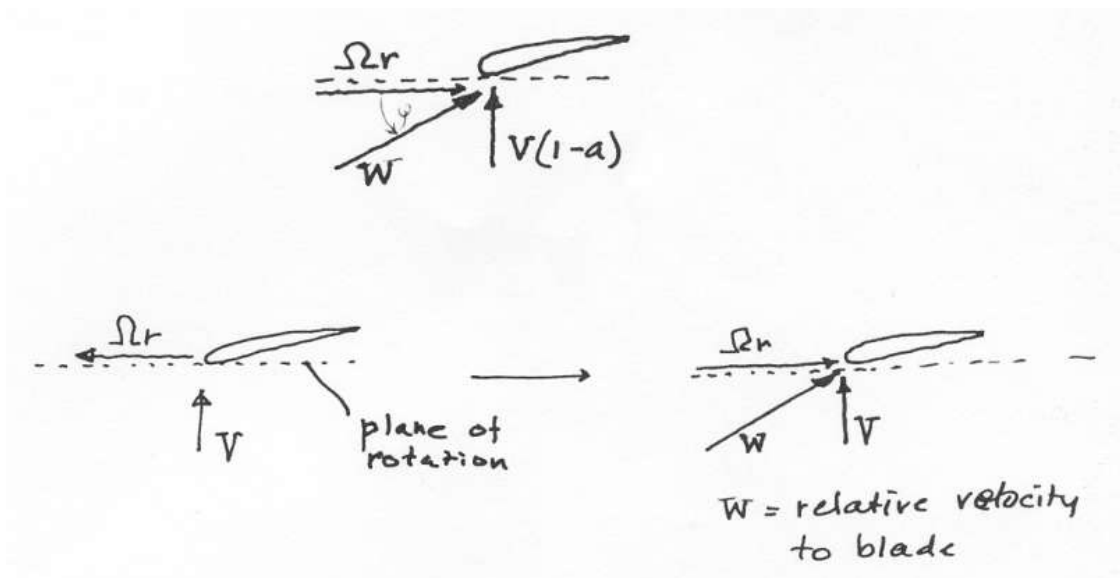


Figure 3.4.a and 3.4.b Showing a blade element and the relative flow to that element.

As Betz model showed the velocity of air at the rotor disk is lower than the wind velocity, by the action of all the rotor blades together. So the relative velocity is slightly different. If the value of the air velocity at the rotor disk is  $V(1 - a)$  (so smaller than  $V$ ) than the relative velocity  $W$  is shown below. (Fig. 3.5.a)

The forces on the blade element result from the relative velocity  $W$  (Fig. 3.5.b).

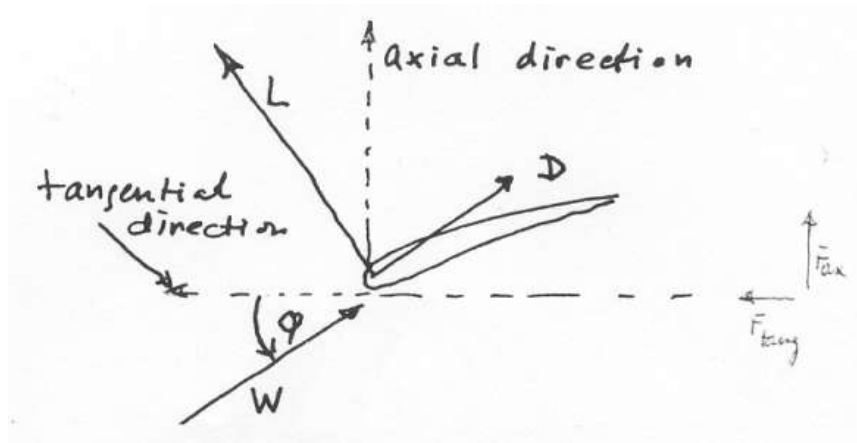


Figure 3.5 Relative flow and forces on a blade element

For a blade element  $\Delta r$ :

$$\Delta F_{tang} = L \sin \varphi - D \cos \varphi$$

$$\Delta F_{axial} = L \cos \varphi + D \sin \varphi$$

$(\Delta F_{tang} \cdot r)$  gives a contribution  $\Delta Q$  to rotor torque. The axial force mainly results from the lift!

$$\text{Now } L = C_L(\alpha) \cdot \frac{1}{2} \rho W^2 \cdot c \cdot \Delta r$$

$$\text{and } D = C_D(\alpha) \cdot \frac{1}{2} \rho W^2 \cdot c \cdot \Delta r$$

$$\alpha = \varphi - \beta$$

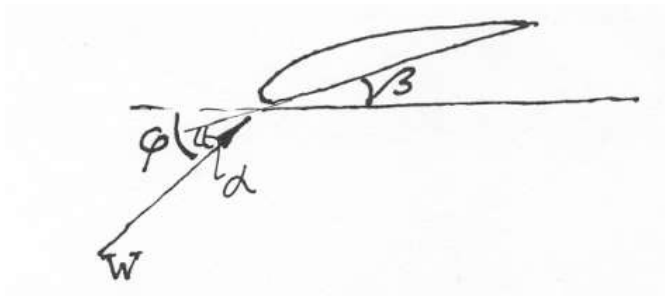


Fig.3.6  $\phi$  is the angle of relative flow to rotor plane

$\beta$  is the angle at which the rotor blade is set

$\alpha$  is angle of attack (see Fig. 2.1).

We could do the calculation for each and every blade element and so find the torque  $Q$  and the axial force  $F_{ax}$  and of course  $P$  (power). Assuming we know  $\Omega$  and  $V$  and  $R$ , we could calculate  $C_Q$ ,  $C_p$  and  $C_{Fax}$ . But it is not so simple! As we do not exactly know the retardation of the flow at the disk ( $V(1 - a)$ ), we do not know  $W$  and  $\phi$ !! And that is what complete rotor theory is about, solving the results from momentum theory and blade element theory all in one go! This, however, lies outside the scope of this course module.

### 3.2.3 Maximum power coefficient

We already explained that according to Betz 16/27 of the power of the wind, that would otherwise pass the rotor area without being disturbed by the rotor, can be extracted by an "actuator disk" or theoretical rotor. A real rotor however is different. Three effects are important, which result in lower power extraction.

#### 1. Wake rotation

The air exerts a torque on the rotor. Vice-versa the rotor exerts a torque on the air. The wake rotates behind the rotor and so the actual velocity of the air in the wake is higher than the velocity if only axial changes are considered. Wake rotation can be considered as a loss!

#### 2. Drag effects

Betz assumes that all the energy extracted by the disk is useful energy. He therefore excludes drag losses. As we can see from equation (3.7a), drag diminishes the resultant torque! The higher  $C_L/C_D$ -value, the less is the influence of drag.

### 3. The effect of finite number of blades

At the tip of the blade, a normal two-dimensional flow cannot be maintained. There is a high negative pressure on the upper side of the airfoil and a positive pressure at the low side (Fig. 2.3). The air will leak around the tip of the blade from the high pressure side to the low pressure side. The lift reduces at the tips. Several methods have been proposed to correct for these tip losses.

The effects of the three types of losses mentioned above have been calculated and are shown in fig. 3.7, 3.8 and 3.9.

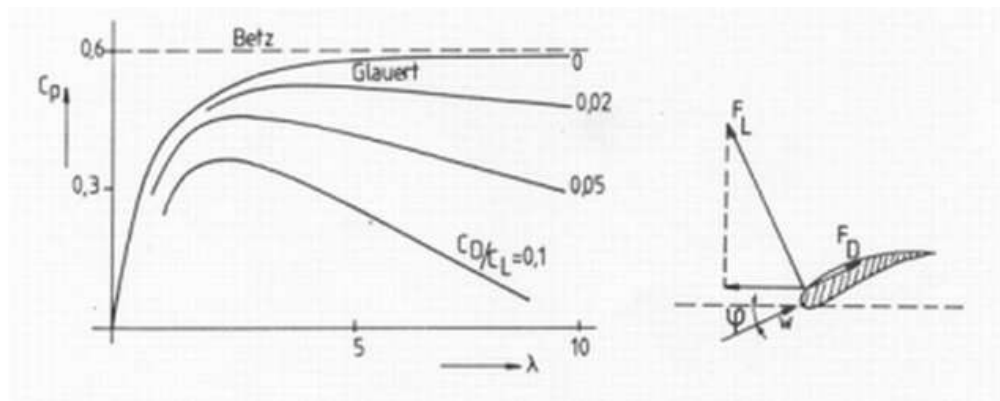


Fig. 3.7 Maximum attainable power coefficient of a rotor with an infinite number of blades designed for a given value of the tip speed ratio  $\lambda$ .

1. Betz value (see section 3.2)

2 Glauerts curve shows effect of wake rotation

3. other curves show combined effect of wake rotation and drag losses.

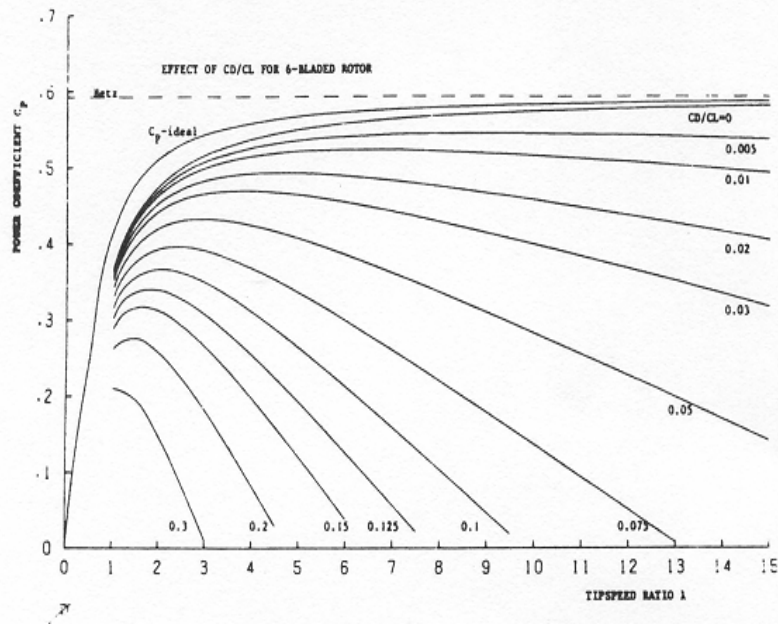


Fig. 3.8 Effect of drag on rotor performance for a six bladed rotor (includes losses owing to wake rotation)

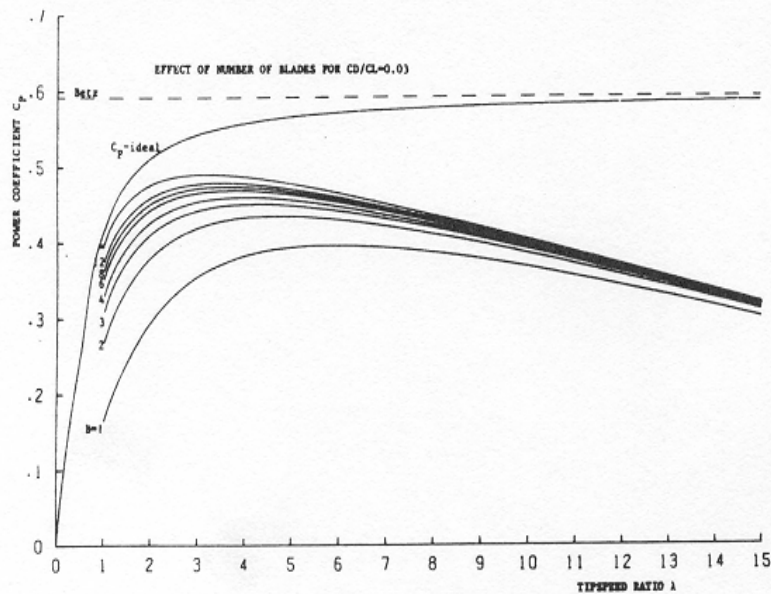


Fig 3.9 Effect of number of blades  $B$  on rotor performance for  $C_D/C_L = 0.03$  (includes losses owing to wake rotation)

Note:

These graphs are not rotor characteristics, i.e. one curve does not represent the characteristics of one type of rotor.

The curves only indicate the **maximum** attainable power coefficient, that could be obtained for a desired tip speed ratio if the rotor is designed for maximum power extraction.

Note:

The curves implicitly show that it is possible to design rotors for different values of the tip speed ratio. That is why we distinguish:

slow running rotors (low  $\lambda$ -value)

fast running rotors (high  $\lambda$ -value).

## 4 REAL ROTOR CHARACTERISTICS

### 4.1 *Power and torque*

We can now try to understand the characteristics of real rotors (Fig.4.1). On the left we have slow running rotors with many blades. On the right we see the fast running rotor with few blades.

Slow running rotors have **HIGH TORQUE, LOW RPM**

Fast running rotors have **LOW TORQUE, HIGH RPM**

Slow running rotors are used for driving piston pumps, that require large torques and run at relatively low speeds.

Fast running rotors are used for electricity generation. Generators run at high speeds and require low torque levels. In general rotor speeds are still too low and (except for very small machines) there is always a gearbox between the rotor and the generator.



$$C_P = \frac{P}{1/2 \rho V^3 \cdot A} \quad C_M = \frac{M}{1/2 \rho V^2 \cdot \pi R^3} \quad \lambda = \frac{\Omega R}{V}$$

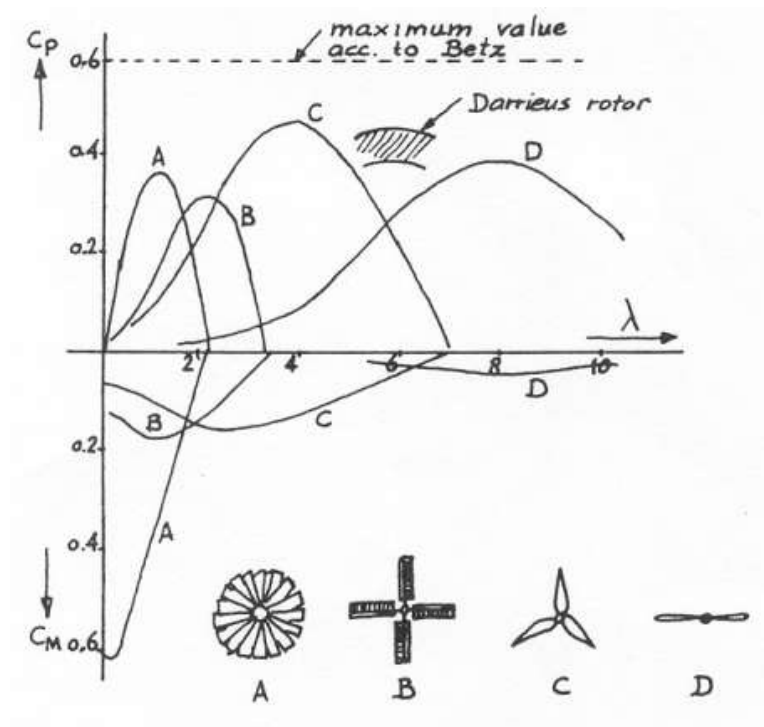


Fig. 4.1 Examples of rotor characteristics

## 4.2 Other force characteristics and rotors in yaw

Up to now we have discussed characteristics of rotors if the direction of the wind is perpendicular to the rotor plane. In practice the wind direction is always fluctuating, so the rotor will never be directed exactly into the wind.

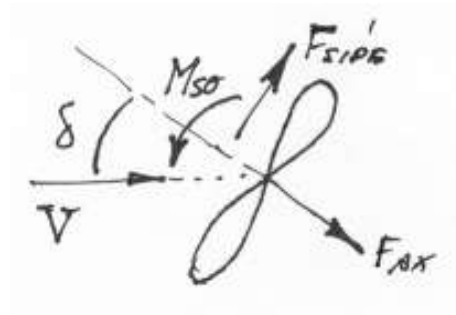
For small WECS, especially wind pumps, the rotor is gradually turned out of the wind at higher wind speeds. This is done for control and safety reasons

- a) to limit the rotational speed of the rotor, the pump etc.
- b) to limit the forces on the rotor and so on the tower.

Turning the rotor out of the wind is called YAWING. If the system to yaw the rotor operates directly by aerodynamic forces of the wind on the rotor and auxiliary vanes, and further by forces of gravity or a spring, such a system is called a passive yaw control system.

Safety and control are treated in detail in two separate modules of this course.

In this section we will discuss a few characteristics of the forces acting on the rotor.



*Fig. 4.2 Top view of rotor. Forces acting on a rotor in yaw*

Consider a rotor in yaw at an angle  $\delta$  to the wind direction. If a rotor is in yaw, then all the forces acting on all the elements of the rotor blades can be combined to two forces acting in the centre of the rotor and a moment (Fig. 4.2).

These two forces are: the axial force  $F_{ax}$  (along the rotor axis) and a side force  $F_s$  (in the rotor plane perpendicular to  $F_{ax}$ ). The moment  $M_{so}$ , which tends to push the rotor back into the wind, is sometimes called the self-orientating moment. This moment is generated by the non-symmetrical distribution of forces on the rotor blades.

We now define dimensionless coefficients for these forces, similar to earlier definitions (equation 3.4, 3.5 and 3.6).

$$C_{Fax}(\lambda, \delta) = \frac{F_{ax}(\delta)}{1/2 \rho V^2 \cdot \pi R^2} \quad \text{axial force coefficient in yaw}$$

$$C_{Fs}(\lambda, \delta) = \frac{F_s(\delta)}{1/2 \rho V^2 \cdot \pi R^2}$$

$$C_{Mso}(\lambda, \delta) = \frac{M_{so}(\delta)}{1/2 \rho V^2 \cdot \pi R^3}$$

and additionally a power coefficient

$$C_p(\lambda, \delta) = \frac{P(\delta)}{1/2 \rho V^3 \cdot \pi R^2}$$

So all these coefficients are not only a function of  $\lambda$  but also of the yaw angle  $\delta$ . The curves shown in Figure 4.1 were  $C_p(\lambda, 0)$  curves, i.e.  $C_p$  as a function of  $\lambda$  with  $\delta = 0$ .

Measurements of forces on wind rotors, especially slow running rotors, are rather scarce. However, the graphs shown, based on measurements at CARDC and TU Eindhoven, are probably typical for slow running rotors.

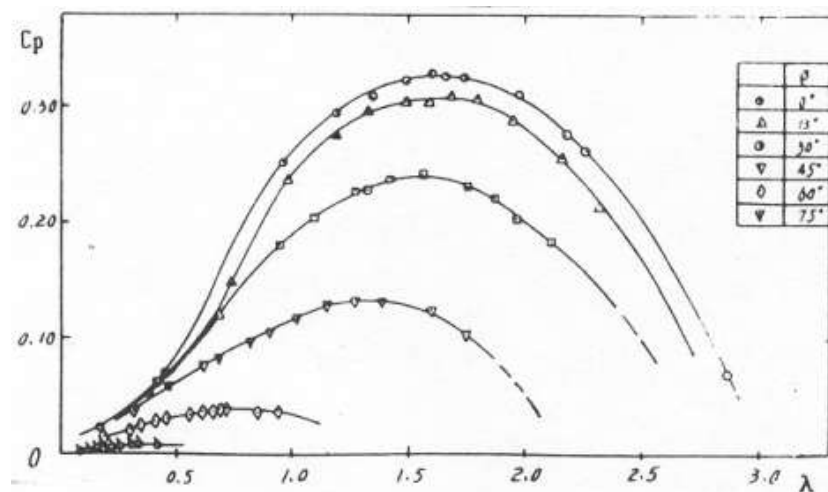


Figure 4.3  $C_p$ - $\lambda$  curves for various yawing angles  $\delta$  of the CWD 2000.

The curve clearly shows that power is reduced by yawing and also the maximum rotor speed is reduced.

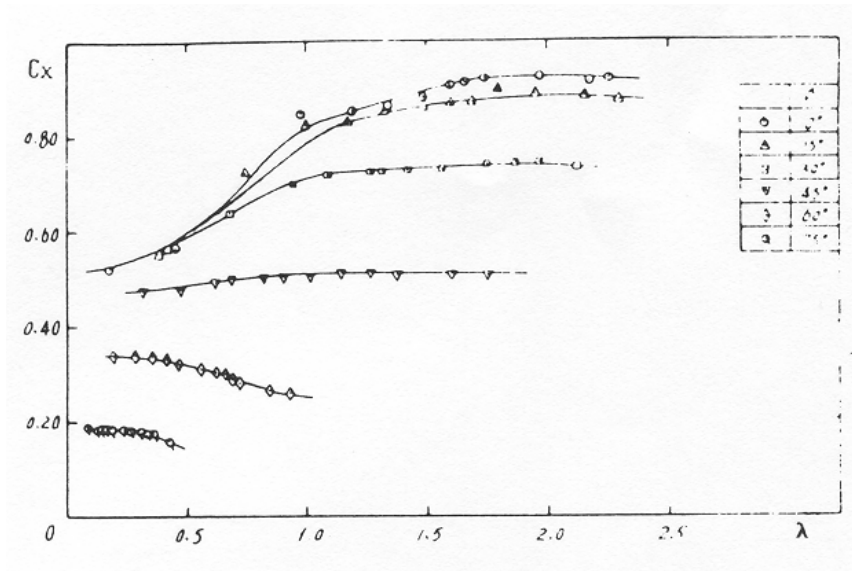
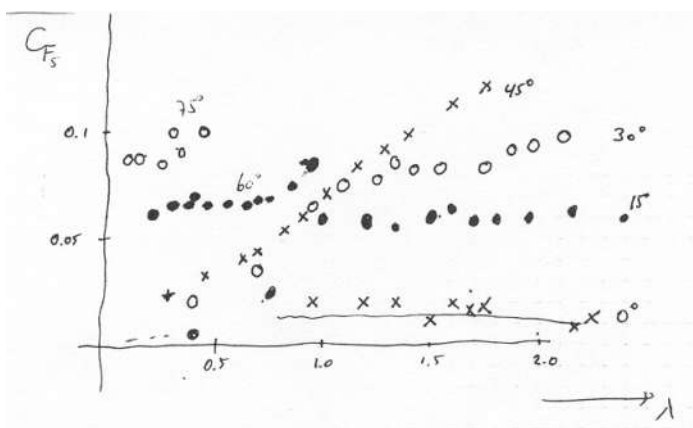


Figure 4.4 The dimensionless axial force ( $C_{Fax}(\lambda, \delta)$ ) for various yawing angles for the CWD2000

Note 1:

$C_{Fax}$  for  $\delta = 0$  is nearly 1 near the design point ( $\lambda = 1.6$ , see Fig. 4.3). Values for fast running rotors say with two blades are more or less the same. Such a value is comparable to the drag coefficient of a solid circular plate of the same size as the rotor! This clearly shows the necessity of safety systems at high windspeeds! So yawing reduces the axial force on the rotor.



*Fig. 4.5 Side force measurements for CWD2000[ ]*

Note: Fig 4.5 shows that at about  $30^\circ$  to  $45^\circ$  the side force becomes relatively important and tends to push the rotor out of the wind.

## Exercises

1. Determine the lift and drag of a NACA 4412 for  $\alpha = 6$  degrees at

40 m/s for a profile of 2 m length and a chord of 0.25 m.

2. Determine the power that a wind rotor of 10 m diameter can extract from the wind? Assume the power coefficient is 0.4 and take a wind speed equal to the average for a region in your country, where the use of wind energy seems favourable. What is the power if the rotor diameter is 1m?

3. A WECS has a rotor of 15 m diameter. Measurements at 6 m/s show that the rotor turns at 60 rpm and that the torque on the axis is

250 Nm. Determine the tip speed ratio at which the rotor is turning; also determine the power coefficient and the torque coefficient. How many blades do you expect the rotor will have approximately?

4. Make a sketch of the relative velocity (relative to the blade element) of a rotor running at a tip speed ratio of 1 (e.g. wind pump) and one of 6 (e.g. electric generation). Explain why the requirements for the aerodynamic properties of the blade to obtain an equal power coefficient, are different.

5. What does an "actuator disk" mean conceptually?

6. A multi-bladed (3 m) rotor of a wind pump and a two bladed (5 m) rotor of a wind generator are located near each other at a test site. The multiblade rotor operates at design ratio tip speed 1 and the rotor of the wind generator at design tip speed 6. What is the ratio of their rotational speeds and estimate the ratio of shaft torque of both machines.

7. Seen from the front a rotor turns clockwise. In which direction does the wake rotate? If two rotors of equal size (one slow runner and one fast runner) operate at the same site, which wake will exhibit more rotation and why?

8. Make a sketch of the lift coefficient of an airfoil.

9. Determine the ratio  $C_D/C_L$  of a NACA 4412 profile for different  $\alpha$  values and sketch the result. What does it tell you?